FE- CRSM) (SEM-I) (CBSGS) 10/5/16

A.M.-I

QP Code : 28579

REVISED COURSE

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **three** questions from question No. 2 to question no. 6.
(3) **Figures** to the right indicate **full** marks.

1. (a) Solve the equation $7\cosh x+8\sinh x = 1$ for real values of x 3

(b) If $z\,(x+y)=(x-y)$ find $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2$ 3

(c) If $u = r^2\cos 2\theta$, $v = r^2 \sin 2\theta$ find $\frac{\partial(u,v)}{\partial(r,\theta)}$ 3

(d) Prove that $\operatorname{Sec}^2 x = 1+x^2+\frac{2x^4}{3}+\ldots\ldots$ 3

(e) Find the rank of the Matrix by reducing it to normal form. 4

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

(f) Find n^{th} derivatives of $\frac{x}{(x-1)(x-2)(x-3)}$ 4

2. (a) If α, β are the roots of the equation $x^2-2\sqrt{3}\cdot x+4=0$ 6
find the value of $\alpha^3+\beta^3$

(b) Examine whether the vecotrs 6

$$X_1=[3 \quad 1 \quad 1],\ X_2=[2 \quad 0 \quad -1]$$
$$X_3=[4 \quad 2 \quad 1]$$

are linearly independent.

(c) (i) State and prove Euler's theorem for a Homogeneous function in two variables. 4

(ii) If $y = x\cos u$ 4
find the value of $x^2u_{xx}+2xy\,u_{xy}+y^2u_{yy}$

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3. (a) Is the following system has trivial or non trivial solution ? Obtain the non trival solution if exist. 6

$$3x_1 + 4x_2 - x_3 - 9x_4 = 0$$
$$2x_1 + 3x_2 + 2x_3 - 3x_4 = 0$$
$$2x_1 + x_2 - 14x_3 - 12x_4 = 0$$
$$x_1 + 3x_2 + 13x_3 + 3x_4 = 0$$

(b) Discuss the stationary points for Maxima and Mininima of 6

$$x^3 + xy^2 - 12x^2 - 2y^2 + 21x + 10$$

(c) (i) If $\tan(x+iy) = a + ib$ prove that $\tanh 2y = \frac{2b}{1+a^2+b^2}$ 4

(ii) Separate into real and imaginary parts of Log (3+4i) 4

4. (a) If $x = u\cos v$, $y = u \sin v$ 6

Prove that $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1$

(b) Show that $\log\left[e^{i\alpha} + e^{i\beta}\right] = \log\left[2\cos\left(\frac{\alpha-\beta}{2}\right)\right] + i\left(\frac{\alpha+\beta}{2}\right)$ 6

(c) (i) Solve the system of equation by Gauss Jordan Method 4

$x + 2y + 6z = 22$, $3x + 4y + z = 26$, $6x - y - z = 19$

(ii) Solve the system of equation by Gauss Siedel Method. 4

Correct upto three decimal.

$$2x - 4y + 49z = 49$$
$$43x + 2y + 25z = 23$$
$$3x + 53y + 3z = 91$$

5. (a) Prove that $\cos^6\theta + \sin^6\theta = \frac{1}{8}[3\cos 4\theta + 5]$ 6

(b) Find the value of a and b 6

if $\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$

(c) (i) If $y = e^x \cos 2x \cos x$ find y_n 4

(ii) If $y = e^{\tan^{-1}x}$ prove that $(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$ 4

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(a) Find non-Singular Matrices P & Q such that, 6

$$A=\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\\ 3 & 0 & 5 & -10\end{bmatrix}$$ is redued to normal form. Also find rank.

(b) If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$ find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$ 6

(c) (i) Fit a straight line to the following data : 4

Year x :	1951	1961	1971	1981	1991
Production y :	10	12	8	10	15

(ii) Fit a second degree parabolic curve to the following data : 4

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9